

A systematic method for converting a polar equation to rectangular form that can be used in many cases

The following method might not be the most efficient for every problem, but it will almost always get the correct answer.

eg.
$$r = \frac{1}{1 + \sin 2\theta}$$

[*] At every stage, multiply by LCDs in order to eliminate fractions immediately.

$$r(1 + \sin 2\theta) = 1$$

[1] Use identities to replace all trigonometric functions involving sums/differences of angles, 2θ , $\frac{1}{2}\theta$ etc. with equivalent functions using only $\sin \theta$, $\cos \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, or $\cot \theta$.

$$r(1 + 2 \sin \theta \cos \theta) = 1$$

[2] Replace all trigonometric functions of θ with their equivalents in x , y and r .

$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \quad \csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}$$

$$r \left(1 + 2 \frac{y}{r} \frac{x}{r} \right) = 1$$

$$r + \frac{2xy}{r} = 1$$

[*]
$$r^2 + 2xy = r$$

[3] Replace r with $(x^2 + y^2)^{\frac{1}{2}}$, and r^2 with $x^2 + y^2$. Simplify exponents.
Square (if necessary) to eliminate fractional exponents.
Expand and collect like terms (if doing so reduces the total number of terms).

$$x^2 + y^2 + 2xy = (x^2 + y^2)^{\frac{1}{2}}$$

$$(x^2 + y^2 + 2xy)^2 = ((x^2 + y^2)^{\frac{1}{2}})^2$$

$$((x + y)^2)^2 = x^2 + y^2$$

$$(x + y)^4 = x^2 + y^2$$

EXPANSION WILL RESULT IN MORE TERMS, SO NO EXPANSION